Answer based on intuition: How many people would you need to have in a room to have a greater than 50% chance that at least two of them share a birthday?

0 - 40	0%
41- 80	0%
81 - 120	0%
121 - 160	0%
161+	0%

Lesson 007
Counting Techniques

September 25, 2023

Counting

- $P(A) = \dfrac{N_A}{N}$ if all events are equally likely.
- How do we find N_A or N

How many possible sequences of heads and tails are there, if you flip a coin 5 times.

$5^2 = 25$

0%

$2^5 = 32$

0%

$2 \times 5 = 10$

0%

5

0%

The Product Rule for Counting

- k choices with n_j options for each choice $j = 1,2,\ldots,k$.
- The **product rule** for counting states that

$$N = n_1 \times n_2 \times \cdots \times n_k = \prod_{j=1}^{k} n_j$$

Product Rule Counting	Total
Number of meals from 4 appetizers, 5 entrees, and 3 desserts.	$4 \times 5 \times 3 = 60$
Combinations in a 4-digit combination lock, with repeating digits allowed.	$10 \times 10 \times 10 \times 10 = 10{,}000$
5 letter words, that start and end in a vowel.	$5 \times 26 \times 26 \times 26 \times 5 = 439{,}400$

A home owner is remodelling and requires both plumbing and electrical contractors. There are 12 plumbing contractors, and 9 electrical contractors. How many ways can the contractors be choosen?

$12^9 = 5,159,780,352$

0%

$9^{12} = 282,429,536,500$

0%

$12 + 9 = 21$

0%

$12 \times 9 = 108$

0%

Combinations	Permutations
Selecting from a set of options when **order does not matter.**	Selecting from a set of options when **order does matter.**

Combinations	Permutations
Forming groups for projects in a class.	Selecting a president, vice president, and treasurer for a club.
Winning lottery numbers.	The combination of a lock.
Possible hands in poker.	Forming words of a set length.

A university has 7 departments, and each department has a student representative. From these representatives, 3 are selected to attend a student conference. To find the number of groups of students attending the conference we would use:

Combinations 0%

Permutations 0%

Combinations

- You wish to select k elements from a set of n elements, in an unordered manner.
- We say "n choose k" and write $\binom{n}{k}$.

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Combinations	Total
Forming a group of 5 from a class of 34 students.	$\binom{34}{5} = \frac{34!}{5!29!} = 278{,}256$
Lottery numbers in a 6-49 style lottery (pick 6 of 49).	$\binom{49}{6} = \frac{49!}{6!43!} = 13{,}983{,}816$
Forming hands of 5 cards from a standard deck of 52.	$\binom{52}{5} = \frac{52!}{5!47!} = 2{,}598{,}960$

Permutations

- You wish to select k elements from a set of n elements, in an ordered manner.
- We say "n permute k" and write $P_{k,n}$.

$$P_{k,n} = \frac{n!}{(n-k)!} = \binom{n}{k} \times k!$$

Permutations	Total
Selecting individuals for five roles from a group of 34 people.	$P_{5,34} = \frac{34!}{29!} = 33{,}390{,}720$
Four digit lock combination with non-repeating digits.	$P_{4,10} = \frac{10!}{6!} = 5040$
Words with non-repeating letters with length 5.	$P_{5,26} = \frac{26!}{21!} = 7{,}893{,}600$

Relating Permutations and Combinations

- Imagine you first select k of the items from the n total, giving you $\binom{n}{k}$.
- Next, you order these items, giving you $P_{k,k} = \frac{k!}{(k-k)!} = k!$

$$N = P_{k,n} = \binom{n}{k} \times k! = \frac{n!}{(n-k)!}$$

In five-card poker, a straight consists of five cards with adjacent denominations, of any suit. If aces can be considered high cards (beyond king) or low cards (before 2), how many possible straights are there?

$\binom{52}{5} = 2,598,960$

0%

$P_{5,13} = 154,540$

0%

$10 \times 4^5 = 10,240$

0%

$4^5 = 1024$

0%

Counting and Probability

- If we count the number of total possible outcomes and the number of outcomes for the event, the ratio of these possibilities is the probability (assuming each event is equally likely).
- A shipment of 25 printers contains 10 laser printers and 15 inkjet printers. If 6 are chosen at random, what is the probability that exactly 3 are laser?

Subsets of 6 printers

$$N = \binom{25}{6}$$

Subsets of 6 printers, with exactly 3 lasers

$$N_A = \binom{10}{3} \times \binom{15}{3}$$

Probability of exactly 3 laser printers

$$P(A) = \frac{N_A}{N} = 0.3083$$

In five-card poker, a straight consists of five cards with adjacent denominations, of any suit. If aces can be considered high cards (beyond king) or low cards (before 2), what is the probability of observing a straight?

$\frac{P_{5,13}}{\binom{52}{5}} = 0.0594$

0%

$\frac{10\times 4^5}{\binom{52}{5}} = 0.00394$

0%

$\frac{10\times 4^5}{P_{5,52}} = 0.0000328$

0%

$\frac{P_{5,13}}{52^5} = 0.82028$

0%

How many possible sets birthdays are there in a room of n individuals (ignore leap years)?

$365 \times n$

0%

$\binom{365}{n}$

0%

$P_{n,365}$

0%

365^n

0%

How many possible sets of birthdays are there, for n distinct individuals, such that each individual has a different birthday?

$365 \times n$

0%

$\binom{365}{n}$

0%

$P_{n,365}$

0%

365^n

0%

What is the probability that, in a group of n distinct individuals, none of them share a birthday?

$\frac{365^n}{P_{n,35}}$

0%

$\frac{\binom{365}{n}}{365^n}$

0%

$\frac{P_{n,365}}{365^n}$

0%

$\frac{\binom{365}{n}}{P_{n,35}}$

0%

All Unique Birthday Probabilities
Probability
1.0
0.8
0.6
0.4
0.2
0.0
0
20
40
60
Number of People in the Room